1. Use logarithms to solve the equation $4^{3x-1} = 3\,(5^x)$, giving your answer correct to 3 decimal places.

[4]

$\ln 4^{3x-1} = \ln 3(5^x)$ M1 *

$(3x-1)\ln 4 = \ln 3 + x\ln 5$ A1

$(3\ln 4 - \ln 5)\,x = \ln 3 + \ln 4$ DM1 *

$\Rightarrow x = \dfrac{\ln 3 + \ln 4}{3\ln 4 - \ln 5}$

$= 0.975$ A1 *

2. Solve the inequality $|x-3| > 2|x+1|$. [4]

$|x-3|^2 > (2|x+1|)^2$ B1

$x^2 - 6x + 9 > 4x^2 + 8x + 4$ M1

$3x^2 + 14x - 5 < 0$

3 -1 A1

1 5

$(3x-1)(x+5) < 0$

$-5 < x < \frac{1}{3}$ A1

3. Showing all necessary working, solve the equation $2\log_2 x = 3 + \log_2(x+1)$, giving your answer correct to 3 significant figures. [5]

$x > 0$,

$\log_2 x^2 = \log_2 8 + \log_2 (x+1)$ M1

M1

$x^2 = 8(x+1)$

$x^2 - 8x - 8 = 0$ A1

$x = \frac{8 \pm \sqrt{64+32}}{2} = 4 \pm 2\sqrt{6}$ M1

96

$4\sqrt{6}$ $\Rightarrow x = 4 + 2\sqrt{6} = 8.90$ A1

4. (i) Prove the identity $\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3$. [4]

$\cos 4\theta = \cos(2\theta \cdot 2)$ B1

$= 2\cos^2 2\theta - 1$ M1

$= 2(2\cos^2\theta - 1)^2 - 1 + 4(2\cos^2\theta - 1)$ A1

$= 2(4\cos^4\theta - 4\cos^2\theta + 1) - 1 + 8\cos^2\theta - 4$

A1

$= 8\cos^4\theta - 3$

(ii) Hence solve the equation $\cos 4\theta + 4\cos 2\theta = 1$ for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$. [3]

$8\cos^4\theta - 3 = 1$ B1

$\cos^4\theta = \frac{1}{2}$

$\cos^2\theta = \frac{\sqrt{2}}{2} = 2^{\frac{1}{2}} \cdot 2^{-1} = 2^{-\frac{1}{2}}$

$\cos\theta = \pm 2^{-\frac{1}{4}}$

B1

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \Rightarrow \cos\theta > 0$

$\Rightarrow \cos\theta = 2^{-\frac{1}{4}} \Rightarrow \theta = \pm 0.572$ B1

5. The polynomial $2x^3+5x^2+ax+b$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x+1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x+2)$ the remainder is 9.

(i) Find the values of a and b. [5]

$$\begin{cases} p(-\tfrac{1}{2}) = 0 \\ p(-2) = 9 \end{cases}$$

B1

M1

$$\begin{cases} 2\cdot\left(-\tfrac{1}{8}\right) + 5\cdot\tfrac{1}{4} - \tfrac{a}{2} + b = 0 \\ -16 + 20 - 2a + b = 9 \end{cases}$$

A1

M1

A1

$\Rightarrow\ a = -4,\ b = -3$

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

$$
\begin{array}{r|l}
 & x^2 + 2x - 3 \\
\hline
2x+1 & 2x^3 + 5x^2 - 4x - 3 \\
 & 2x^3 + x^2 \\
\hline
 & 4x^2 - 4x - 3 \\
 & 4x^2 + 2x \\
\hline
 & -6x - 3
\end{array}
$$

A1

M1

$p(x) = (2x+1)(x^2+2x-3)$

$= (2x+1)(x+3)(x-1)$ A1

$p(-\frac{1}{2}) = 0$ B1

$p(-2) = 9$ M1

M1

M1

$2(-\frac{1}{8}) +$ $= p$

$-16 + 20$ 9

$\Rightarrow a = -4, b = -3$ A1

6. (i) Express $\sqrt{6}\cos\theta + \sqrt{10}\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

$R^2 = (\sqrt{6})^2 + (\sqrt{10})^2 = 16$

B1

$\Rightarrow R = 4$

$4\left(\frac{\sqrt{6}}{4}\cos\theta + \frac{\sqrt{10}}{4}\sin\theta\right)$

M1

$\Rightarrow \cos\alpha = \frac{\sqrt{6}}{4}$

$\sin\alpha = \frac{\sqrt{10}}{4}$

A1

$\Rightarrow \alpha = \cancel{0.91}\ 52.24°$

(ii) Hence, in each of the following cases, find the smallest positive angle θ which satisfies the equation

(a) $\sqrt{6}\cos\theta + \sqrt{10}\sin\theta = -4$, [2]

$4\cos(\theta - 52.24^\circ) = -4$

$\theta - 52.24^\circ = 180^\circ + k \cdot 360^\circ$

M1

$\Rightarrow \theta = 232.24^\circ + k \cdot 360^\circ$

$\theta_{min} = 232.2^\circ$ A1

(b) $\sqrt{6}\cos\frac{1}{2}\theta + \sqrt{10}\sin\frac{1}{2}\theta = 3$, [4]

$4\cos(\frac{1}{2}\theta - 52.24^\circ) = 3$ M1

$\frac{1}{2}\theta - 52.24^\circ = \pm\cos^{-1}(\frac{3}{4}) + k \cdot 360^\circ$ A1

$\theta = 187.3^\circ + k \cdot 720^\circ$ / $\theta = 21.7^\circ + k \cdot 720^\circ$

M1

$\theta_{min} = 21.7^\circ$ A1

7. Let $f(x) = \frac{5x^2 + x + 6}{(3 - 2x)(x^2 + 4)}$.

(i) Express $f(x)$ in partial fractions. [5]

$$\frac{5x^2+x+6}{(3-2x)(x^2+4)} = \frac{A}{3-2x} + \frac{Bx+C}{x^2+4}$$ B1

$$5x^2+x+6 = A(x^2+4) + (Bx+C)(3-2x)$$

3

Let $x = \frac{3}{2}$ M1

$$5 \cdot \frac{9}{4} + \frac{3}{2} + 6 = A\left(\frac{9}{4}+4\right)$$

$\Rightarrow A = 3$ A1

Let $x=0$

$$6 = 3(4) + C(3)$$ A1

$\Rightarrow C = -2$

Let $x=1$

$$5+1+6 = 3(5) + (B-2)(1)$$

$\Rightarrow B = -1$ A1

$$f(x) = \frac{3}{3-2x} + \frac{-x-2}{x^2+4}$$

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

$$\left(1+\left(-\frac{2}{3}x\right)\right]^{-1} - \frac{1}{4}(x+2)\left(1+\frac{x^2}{4}\right)^{-1}$$ M1

$$= 1+\frac{2}{3}x+\left(-\frac{2}{3}x\right)^2 - \frac{1}{4}(x+2)\left(1-\frac{x^2}{4}\right)$$ A1 A1

$$= 1+\frac{2}{3}x+\frac{4}{9}x^2 - \frac{1}{4}\left(x+2-\frac{x^2}{2}\right)$$ M1

$$= \left(1-\frac{1}{2}\right)+\left(\frac{2}{3}-\frac{1}{4}\right)x+\left(\frac{4}{9}+\frac{1}{8}\right)x^2+\cdots$$

$$= \frac{1}{2}+\frac{5}{12}x+\frac{41}{72}x^2+\cdots$$ A1

THE END